

Illustration of Basic Tours Used in Optimal Algorithms for an Extended Version of the Knight's Tour Problem

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Abstract: This document contains supplementary material to our paper “Optimal Algorithms for an Extended Version of the Knight's Tour Problem” [1] that is concerned with determining optimal solutions for the Traveling Salesman Problem with Forbidden Neighborhood (TSPFN) with $r = 2$. In particular we depict a selection of optimal TSPFN tours with $r = 2$ as well as double loops and structured and stretched knight's tours on various $m \times n$ grids with $m, n \leq 12$.

Optimal tours on $2 \times n$ grids with $n \leq 12$

1	8	5	2	9
6	3	10	7	4

Figure 1.1: An optimal TSPN tour on the 2×5 grid.

1	3	7	10	2	4
6	9	12	5	8	11

Figure 1.2: An optimal TSPN tour on the 2×6 grid.

1	12	7	2	11	6	3
8	5	14	9	4	13	10

Figure 1.3: An optimal TSPN tour on the 2×7 grid.

1	11	8	2	13	7	4	14
9	3	16	10	5	15	12	6

Figure 1.4: An optimal TSPN tour on the 2×8 grid.

1	12	9	2	13	6	17	14	7
10	3	18	11	4	15	8	5	16

Figure 1.5: An optimal TSPN tour on the 2×9 grid.

1	18	15	2	19	14	7	10	13	6
16	3	20	17	4	9	12	5	8	11

Figure 1.6: An optimal TSPN tour on the 2×10 grid.

1	14	11	2	15	10	5	18	9	6	19
12	3	22	13	4	21	16	7	20	17	8

Figure 1.7: An optimal TSPN tour on the 2×11 grid.

1	15	12	2	16	11	5	22	20	6	9	19
13	3	24	14	4	23	17	10	8	18	21	7

Figure 1.8: An optimal TSPN tour on the 2×12 grid.

Optimal tours on $3 \times n$ grids with $n = 9, 10, 11, 12$

1	25	4	15	18	21	8	13	10
3	16	23	26	5	14	11	20	7
24	27	2	17	22	19	6	9	12

Figure 2.1: Optimal TSPN tour for the 3×9 grid.

1	4	29	20	7	22	25	16	13	10
28	19	2	5	26	17	8	11	24	15
3	30	27	18	21	6	23	14	9	12

Figure 2.2: Closed knight's tour on the 3×10 grid.

30	33	2	27	8	19	6	21	10	13	16
3	26	29	32	5	24	9	18	15	22	11
1	31	4	25	28	7	20	23	12	17	14

Figure 2.3: Optimal TSPN tours for the 3×11 grid.

1	4	7	28	31	34	21	26	23	18	15	12
6	29	2	35	8	27	32	19	10	13	24	17
3	36	5	30	33	20	9	22	25	16	11	14

Figure 2.4: Closed knight's tour on the 3×12 grid.

Optimal tours on $m \times n$ grids with $5 \leq m, n \leq 10$

Grids with both m and n odd

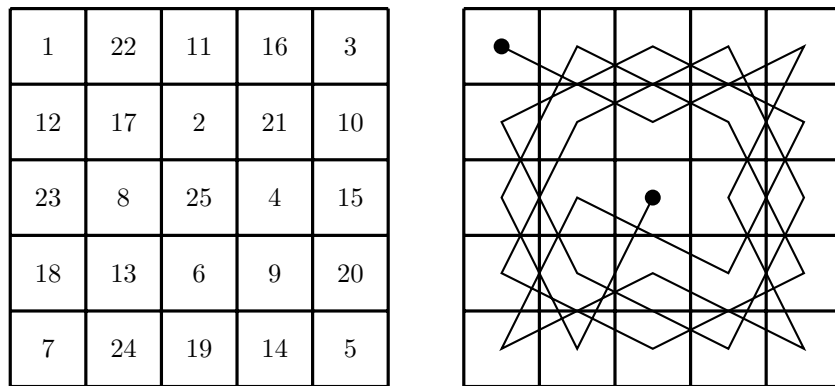


Figure 3.1: Optimal TSPN tour on the 5×5 grid.

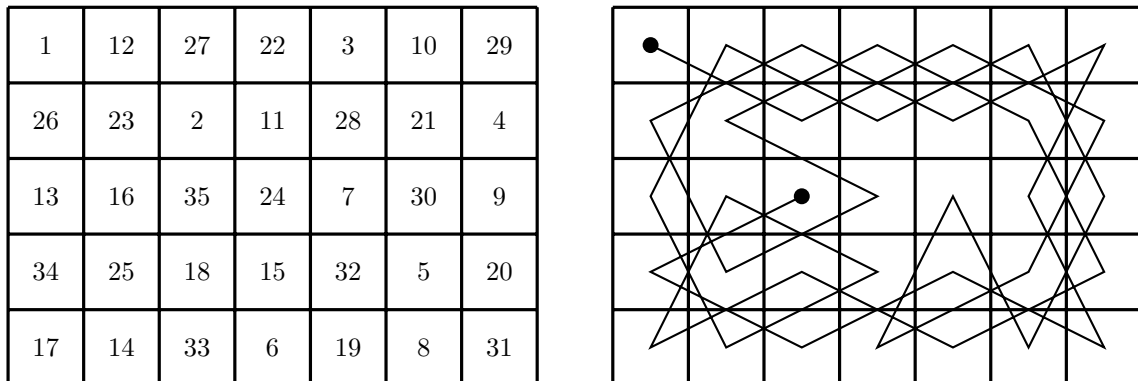


Figure 3.2: Optimal TSPN tour on the 5×7 grid.

33	12	29	42	3	14	27
30	43	32	13	28	5	2
11	34	41	4	1	26	15
44	31	10	25	40	19	6
35	24	49	46	9	16	39
48	45	22	37	18	7	20
23	36	47	8	21	38	17

Figure 3.3: An optimal TSPN tour on the 7×7 grid.

1	42	21	26	3	40	35	6	11
22	27	2	41	20	5	10	39	34
43	18	45	4	25	36	31	12	7
28	23	16	19	30	9	14	33	38
17	44	29	24	15	32	37	8	13

Figure 3.4: An optimal TSPN tour on the 5×9 grid.

31	62	27	40	1	16	23	52	45
28	39	30	37	26	51	44	15	22
61	32	63	2	41	24	17	46	53
6	29	38	25	36	43	50	21	14
33	60	7	42	3	18	11	54	47
8	5	58	35	10	49	56	13	20
59	34	9	4	57	12	19	48	55

Figure 3.5: An optimal TSPN tour on the 7×9 grid.

53	64	79	58	1	8	17	32	41
80	59	54	63	78	31	40	9	18
65	52	81	2	57	16	7	42	33
60	55	4	77	62	39	30	19	10
51	66	61	56	3	6	15	34	43
68	27	72	5	76	29	38	11	20
73	50	67	28	71	14	23	44	35
26	69	48	75	24	37	46	21	12
49	74	25	70	47	22	13	36	45

Figure 3.6: An optimal TSPN tour on the 9×9 grid.

Grids with m or n even

1	10	5	22	25	16
4	21	2	17	6	23
11	30	9	24	15	26
20	3	28	13	18	7
29	12	19	8	27	15

Figure 3.7: Structured closed knight's tour on the 5×6 grid.

1	10	33	36	3	12	19	24
32	35	2	11	20	25	4	13
9	40	29	34	37	16	23	18
28	31	38	7	26	21	14	5
39	8	27	30	15	6	17	22

Figure 3.8: Structured closed knight's tour on the 5×8 grid.

1	10	5	22	25	16	45	40	35	30
4	21	2	17	6	23	26	31	44	39
11	50	9	24	15	46	41	36	29	34
20	3	48	13	18	7	32	27	38	43
49	12	19	8	47	14	37	42	33	28

Figure 3.9: Structured closed knight's tour on the 5×10 grid.

1	24	27	8	3	16
26	9	2	15	28	7
23	36	25	6	17	4
10	33	18	21	14	29
35	22	31	12	5	20
32	11	34	19	30	13

Figure 3.10: Structured closed knight's tour on the 6×6 grid.

1	26	11	22	9	40	31
12	23	42	25	30	21	8
27	2	29	10	41	32	39
16	13	24	33	36	7	20
3	28	15	18	5	38	35
14	17	4	37	34	19	6

Figure 3.11: Structured closed knight's tour on the 6×7 grid.

1	18	43	36	3	16	13	34
42	37	2	17	44	35	4	15
19	48	9	38	5	14	33	12
8	41	22	47	10	45	30	27
23	20	39	6	25	28	11	32
40	7	24	21	46	31	26	29

Figure 3.12: Structured closed knight's tour on the 6×8 grid.

1	34	9	14	49	32	29	26	47
10	15	2	33	8	13	48	31	28
35	54	11	50	3	30	27	46	25
16	51	18	7	12	5	22	43	40
19	36	53	4	21	38	41	24	45
52	17	20	37	6	23	44	39	42

Figure 3.13: Structured closed knight's tour on the 6×9 grid.

1	20	59	24	3	22	11	14	5	44
58	35	2	21	40	25	4	43	10	13
19	60	39	36	23	42	15	12	45	6
34	57	32	41	26	37	28	9	48	51
31	18	55	38	29	16	53	50	7	46
56	33	30	17	54	27	8	47	52	49

Figure 3.14: Structured closed knight's tour on the 6×10 grid.

1	22	43	32	3	20	45	34
42	31	2	21	44	33	6	19
23	56	11	4	17	28	35	46
52	41	30	27	12	5	18	7
55	24	53	10	29	16	47	36
40	51	26	13	38	49	8	15
25	54	39	50	9	14	37	48

Figure 3.15: Structured closed knight's tour on the 7×8 grid.

1	18	69	32	3	20	47	24	5	22
68	65	2	19	60	31	4	21	38	25
17	70	63	66	33	48	41	46	23	6
64	67	52	59	42	61	30	37	26	39
53	16	55	62	49	34	45	40	7	10
56	51	14	43	58	29	12	9	36	27
15	54	57	50	13	44	35	28	11	8

Figure 3.16: Structured closed knight's tour on the 7×10 grid.

1	50	25	44	63	48	23	18
26	43	64	49	24	19	62	47
51	2	27	20	45	12	17	22
42	29	10	3	14	21	46	61
9	52	5	28	11	16	13	36
30	41	8	15	4	35	60	57
53	6	39	32	55	58	37	34
40	31	54	7	38	33	56	59

Figure 3.17: Structured closed knight's tour on the 8×8 grid.

1	70	37	56	15	68	39	54	13
36	57	72	69	38	55	14	67	40
71	2	35	16	43	4	41	12	53
58	19	44	3	30	17	52	7	66
45	34	29	18	5	42	11	24	51
28	59	20	31	48	25	6	65	8
33	46	61	26	21	10	63	50	23
60	27	32	47	62	49	22	9	64

Figure 3.18: Structured closed knight's tour on the 8×9 grid.

1	22	47	8	63	78	45	10	41	58
48	7	2	79	46	9	64	59	44	11
23	80	21	62	77	66	75	42	57	40
20	49	6	3	74	55	60	65	12	43
5	24	71	54	61	76	67	56	39	30
50	19	4	73	70	53	38	31	34	13
25	72	17	52	27	68	15	36	29	32
18	51	26	69	16	37	28	33	14	35

Figure 3.19: Structured closed knight's tour on the 8×10 grid.

1	68	57	82	13	4	55	80	51	20
58	83	2	89	56	81	14	19	54	79
67	90	69	12	3	88	5	52	21	50
70	59	84	87	6	15	18	25	78	53
37	66	7	16	11	86	9	22	49	30
60	71	36	85	8	17	26	29	24	77
65	38	63	42	27	10	23	46	31	48
72	61	40	35	74	43	28	33	76	45
39	64	73	62	41	34	75	44	47	32

Figure 3.20: Structured closed knight's tour on the 9×10 grid.

1	26	71	30	3	24	37	20	5	22
72	29	2	25	70	33	4	23	36	19
27	100	73	82	31	38	47	34	21	6
60	85	28	69	74	81	32	39	18	35
99	76	61	86	83	48	67	46	7	40
62	59	84	75	68	87	80	41	50	17
77	98	63	12	79	66	49	88	45	8
58	95	78	65	54	13	42	51	16	89
97	64	93	56	11	52	91	14	9	44
94	57	96	53	92	55	10	43	90	15

Figure 3.21: Structured closed knight's tour on the 10×10 grid.

Stretched knight's tours on $m \times 6$ grids with $5 \leq m \leq 10$

1	14	29	20	3	12
30	21	2	13	28	19
15	8	17	24	11	4
22	25	6	9	18	27
7	16	23	26	5	10

Figure 4.1: Stretched knight's tour on the 5×6 grid.

1	18	9	26	3	20
36	25	2	19	10	27
17	8	35	28	21	4
24	31	22	13	34	11
7	16	29	32	5	14
30	23	6	15	12	33

Figure 4.2: Stretched knight's tour on the 6×6 grid.

1	14	39	26	3	16
42	25	2	15	36	27
13	38	29	40	17	4
30	41	24	37	28	35
9	12	31	20	5	18
32	23	10	7	34	21
11	8	33	22	19	6

Figure 4.3: Stretched knight's tour on the 7×6 grid.

1	14	47	38	25	16
48	41	2	15	22	37
13	46	39	26	17	24
40	3	42	23	36	21
43	12	45	20	27	18
6	9	4	33	30	35
11	44	7	28	19	32
8	5	10	31	34	29

Figure 4.4: Stretched knight's tour on the 8×6 grid.

1	26	21	16	3	28
54	15	2	27	20	17
25	22	53	18	29	4
14	41	24	51	34	19
23	52	35	42	5	30
40	13	50	31	46	33
49	10	47	36	43	6
12	39	8	45	32	37
9	48	11	38	7	44

Figure 4.5: Stretched knight's tour on the 9×6 grid.

1	50	59	20	39	48
60	17	2	49	36	21
51	58	19	40	47	38
18	3	16	37	22	35
5	52	57	24	41	46
56	15	4	45	34	23
53	6	55	42	25	44
14	9	12	33	28	31
11	54	7	30	43	26
8	13	10	27	32	29

Figure 4.6: Stretched knight's tour on the 10×6 grid.

Double loops on 5×4 , 7×4 and 9×4 grids

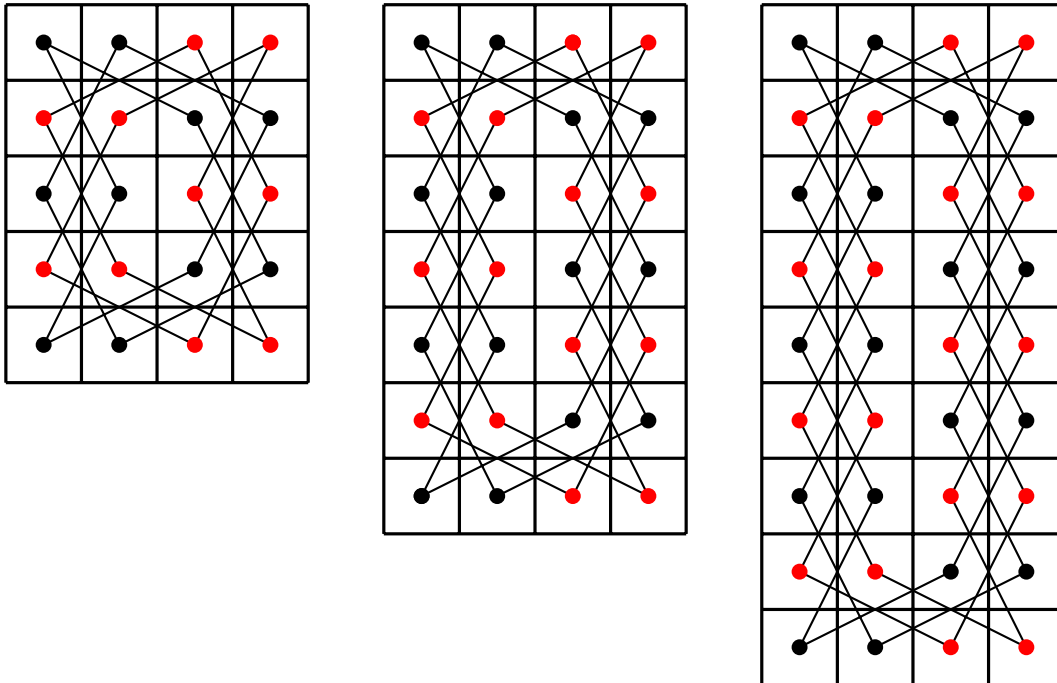


Figure 5.1: Double loops on the 5×4 , 7×4 and 9×4 grids.

References

- [1] M. Firstein, A. Fischer, and P. Hungerländer. Optimal algorithms for an extended version of the knight's tour problem. Technical report, Alpen-Adria Universität Klagenfurt, Mathematics, Optimization Group, TR-ARUK-M-O-17-04, April 2017.